OCR
Oxford Cambridge and RSA

Friday 15 June 2018 – Afternoon

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

DC (LK) 131833/3

1 In this question, you may assume that $t^k e^{-t} \to 0$ as $t \to \infty$ for any constant k.

The differential equation $4\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 9x = f(t)$ is to be solved for $t \geqslant 0$.

Firstly consider the case $f(t) = 9t^2 - 3t - 1$.

(i) Find the general solution for x in terms of t. **[9]**

You are given that $x = 5$ and $\frac{dx}{dt} = 0$ when $t = 0$.

(ii) Find the particular solution. **[4]**

(iii) Show that x is positive for all values of $t \geqslant 0$. **[3]**

Now consider the case $f(t) = -48\sin 2t - 14\cos 2t$.

(iv) Find the general solution for x in terms of t. **[6]**

(v) Describe the behaviour of x for large values of t. **[2]**

2 Take g as 10 in this question.

A particle P of mass 0.1 kg is in a liquid and is projected vertically downwards. At time t s, the velocity of P is $v\,\text{m s}^{-1}$ and the depth of P below its point of projection, O, is x m. The only forces on P are its weight and a resistance force R N. A scientist investigates two different models for R.

In the first model, the resistance is given by $R = 0.2v$ and the initial speed of P is $2\,\text{m s}^{-1}$.

(i) Use this information to form a differential equation involving v and t. Solve the differential equation to show that $v = 5 - 3e^{-2t}$. **[7]**

(ii) Sketch the graph of v against t. **[2]**

(iii) Find an expression for x in terms of t and hence find the depth of P below O when its speed is three-quarters of its terminal speed. **[7]**

In the second model, the resistance is given by $R = 0.0625v^2$ and the initial speed of P is again $2\,\text{m s}^{-1}$.

(iv) Find v in terms of x. **[6]**

(v) State the terminal speed of P and find the depth of P below O when its speed is three-quarters of its terminal speed. **[2]**

3 **(a)** A curve in the x-y plane satisfies the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{2y}{x} = x^k \sin 2x$,

where k is a constant and $x > 0$.

Firstly consider the case $k = 3$.

(i) Find the general solution for y in terms of x. **[7]**

(ii) Given that $y = 0$ when $x = \frac{1}{4}\pi$, find the exact value of y when $x = \frac{1}{2}\pi$. **[4]**

Now consider the case $k = 2.5$.

(iii) Use Euler's method, with a step length of 0.1 and initial conditions $y = 0$ when $x = 0.5$, to estimate y when $x = 0.8$. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$. **[5]**

(b) Solutions of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 - y$ are to be investigated using a tangent field.

(i) Show that the isocline for which $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ is a parabola. State the coordinates of its turning point. **[2]**

(ii) In your Answer Book, sketch on the given axes the isoclines for the cases $\frac{\mathrm{d}y}{\mathrm{d}x} = m$ for $m = 0, \pm 1, \pm 2$. Use these isoclines to draw a tangent field. **[3]**

(iii) Sketch the solution curve through (0, 1) and the solution curve through (1, 0). **[3]**

4 The simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 7x + 2y + 13\mathrm{e}^{4t},$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -9x + y + \mathrm{e}^{7t}$$

are to be solved.

(i) Eliminate x to obtain a second order differential equation for y in terms of t. Hence find the general solution for y. **[12]**

(ii) Given that $y = -3$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 60$ when $t = 0$, find the particular solution for y. **[4]**

(iii) Find the corresponding particular solution for x. **[2]**

(iv) Find the smallest positive value of t for which $y = 0$. **[4]**

(v) Show that $\frac{y}{x} \to 0$ as $t \to \infty$. **[2]**

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Friday 15 June 2018 – Afternoon

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:
- Question Paper 4758/01 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4758 01

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

1 (i)	

1 (ii)

1 (iii)

1 (iv)	
1 (v)	

2 (i)

2 (ii)

2 (iii)

(answer space continued on next page)

2 (iii)	**(continued)**

2 (iv)

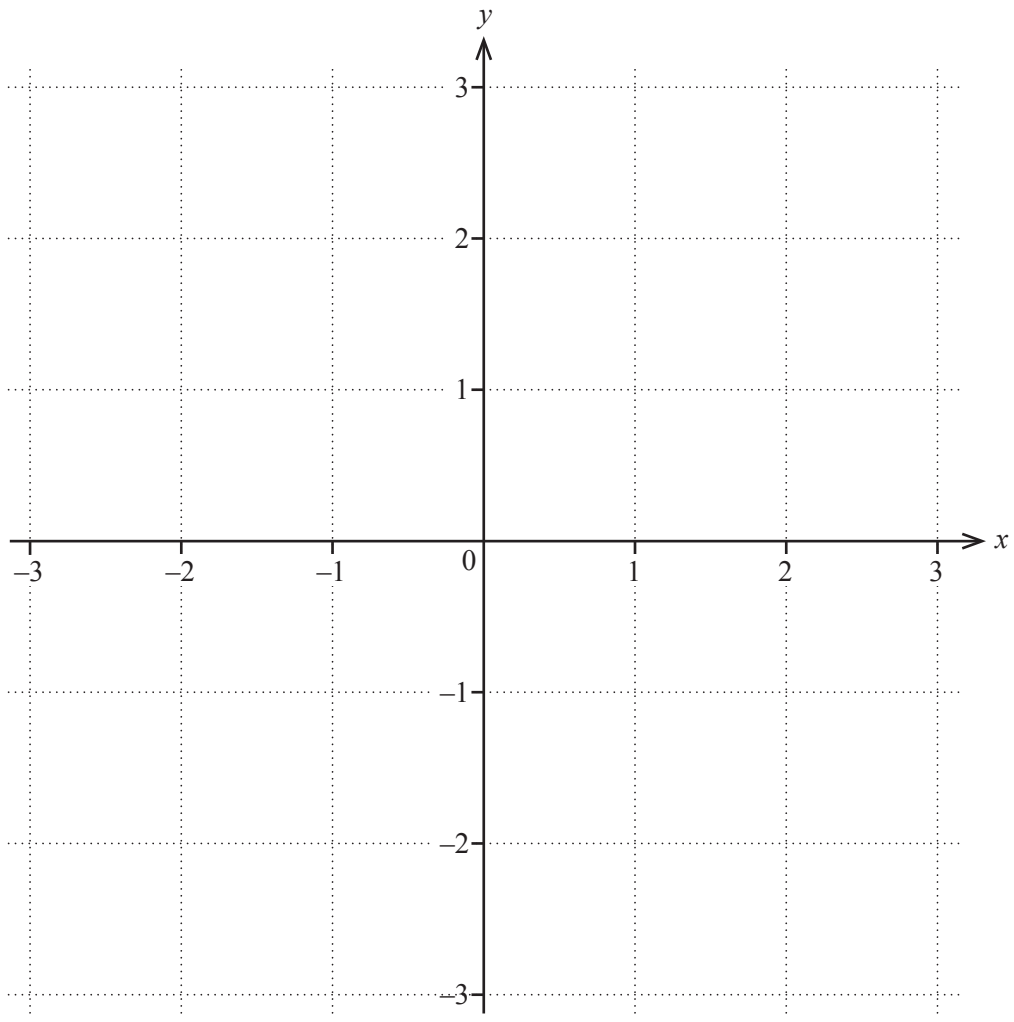
2 (v)

3 (a)(i)	

3 (a)(ii)	
3 (a)(iii)	

3 (b)(i)

3 (b) (ii) & (iii)



A spare copy of this diagram can be found on page 16.

4 (i)	
	(answer space continued on next page)

4 (i)	**(continued)**
4 (ii)	

4 (iii)

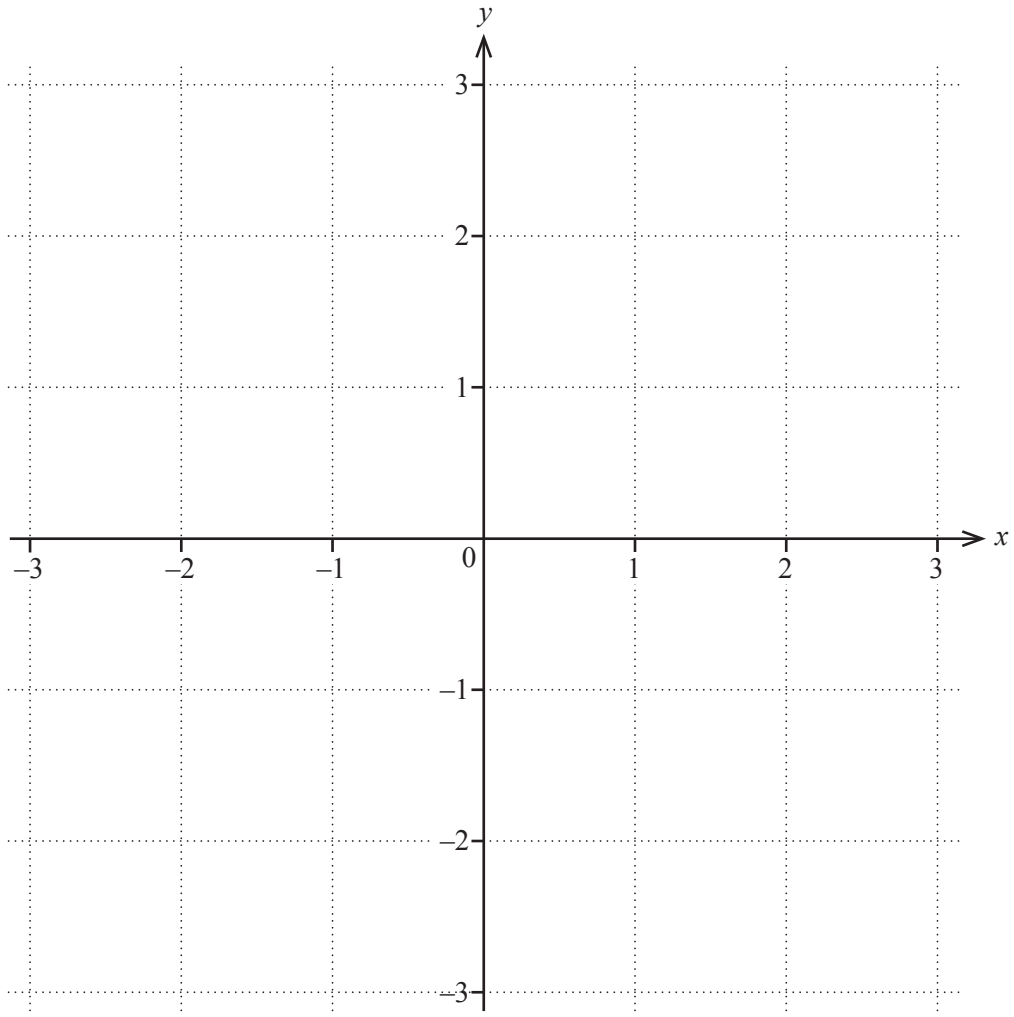
4 (iv)

(answer space continued on next page)

4 (iv)	**(continued)**
4 (v)	

3 (b) (ii) & (iii)

Spare copy of diagram for questions 3 (b) (ii) and 3 (b) (iii)



OCR
Oxford Cambridge and RSA

OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4758**: Differential Equations

Advanced GCE

Mark Scheme for June 2018

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.
ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an

answer given. Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate's script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	Guidance	
1.	(i)		Auxiliary equation: $4m^2+12m+9=0$	M1		
			$m=-\frac{3}{2}$ (twice)	A1		
			CF: $x=(At+B)\mathrm{e}^{-\frac{3}{2}t}$	F1	From their roots	
			PI: $x=Ct^2+Dt+E$	B1	Correct form	
			$\dot{x}=2Ct+D, \quad \ddot{x}=2C$ $9C=9$	M1	Differentiate and substitute	
			$24C+9D=-3$	M1	Compare coefficients	
			$8C+12D+9E=-1$	M1	Attempt to solve	
			$C=1, D=-3, E=3$	A1		
			GS: $x=(At+B)\mathrm{e}^{-\frac{3}{2}t}+t^2-3t+3$	F1	CF with 2 arb constants + PI	
				[9]		
	(ii)		$x=5, t=0: \ B=2$	M1	Use condition	
			$\dot{x}=e^{-\frac{3t}{2}}\left(A-\frac{3}{2}(At+B)\right)+2t-3$	M1	Differentiate GS using product rule	
			$\dot{x}=0, t=0: \ A=6$	M1	Use condition	
			$x=(6t+2)e^{-\frac{3t}{2}}+t^2-3t+3$	A1	cao	
				[4]		
	(iii)		Exponential term never negative (for $t\geq 0$)	B1		
			PI = $\left(t-\frac{3}{2}\right)^2+\frac{3}{4}$	M1	Complete the square, or use discriminant or differentiate as far as finding *t* or any appropriate method	
			Always positive	A1	Show PI positive e.g. identify minimum *x* at ¾	
				[3]		

Question			Answer	Marks	Guidance	
	(iv)		PI: $x = A\cos 2t + B\sin 2t$ $\dot{x} = -2A\sin 2t + 2B\cos 2t$ $\ddot{x} = -4A\cos 2t - 4B\sin 2t$ $24B - 7A = -14;\ -7B - 24A = -48$ $A = 2, B = 0$ $x = 2\cos 2t$ GS: $x = (At + B)\mathrm{e}^{-\frac{3}{2}t} + 2\cos 2t$	B1 M1 M1 M1 A1 F1 **[6]**	Correct form Differentiate and substitute Compare coefficients Attempt to solve CF correct of FT from **(i)** + PI from **(iv)** CF with 2 arb constants + PI	
	(v)		First term $\to 0$ as $t \to \infty$ AND oscillations Amplitude 2	B1 B1FT **[2]**	Implied by $'x \to 2\cos 2t$ as $t \to \infty'$ FT their PI from (iv)	
2.	(i)		Downwards + ve: $0.1\dfrac{\mathrm{d}v}{\mathrm{d}t} = 0.1g - 0.2v$ $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 2(5 - v)$ or $\frac{\mathrm{d}v}{\mathrm{d}t} + 2v = 10$ **Method 1:** $\dfrac{\mathrm{d}v}{5 - v} = 2\mathrm{d}t$ $-\ln(5 - v) = 2t + A$ $t = 0, v = 2:\quad A = -\ln 3$ $v = 5 - 3\mathrm{e}^{-2t}$ AG **Method 2:** IF: $\mathrm{e}^{\int 2\mathrm{d}t} = \mathrm{e}^{2t}$ $v\mathrm{e}^{2t} = 5\mathrm{e}^{2t} + A$ $v = A\mathrm{e}^{-2t} + 5$	M1 A1 M1 A1 M1 M1 A1 **[7]** B1 M1 M1	Condone $g = 9.8$ for first 6 marks. Use N2L, allow sign errors cao any form Separate variables and attempt to integrate Use condition Rearrange cao Multiply by IF and attempt to integrate both sides Divide both sides by IF	

Question			Answer	Marks	Guidance	
			$t=0, v=2: \quad A=-3$ $v=5-3\mathrm{e}^{-2t}$ **Method 3:** CF: $m=-2$: $A\mathrm{e}^{-2t}$ PI: $x=5$ GS: $y=A\mathrm{e}^{-2t}+5$ $t=0, v=2: \quad A=-3$ $v=5-3\mathrm{e}^{-2t}$	M1 A1 **[7]** M1 B1 M1 M1 A1 **[7]**	Use condition cao Find CF Find PI Find GS (their CF + their PI) Use condition cao	
	(ii)		Correct shape starting at (2, 0) Asymptote at $v=5$	B1 B1 **[2]**		
	(iii)		$x=5t+\frac{3}{2}\mathrm{e}^{-2t}+B$ $x=0, t=0 \qquad B=-\frac{3}{2}$ $x=5t+\frac{3}{2}\mathrm{e}^{-2t}-\frac{3}{2}$ $\frac{3}{4}$ of terminal speed is $\frac{15}{4}$ m s^{-1} $e^{-2t}=\frac{5}{12}$ t = 0.437734… so 0.438 (3 s.f.) x = 1.31367… so 1.31 (3 s.f.)	M1 M1 A1 B1 M1 A1 A1 **[7]**	Condone g = 9.8 for M1M1A0B1M1A0A0 max Integrate Use condition cao cao	

Question			Answer	Marks	Guidance	
	(iv)		$v\frac{\mathrm{d}v}{\mathrm{d}x}=10-0.625v^2$	B1	Any correct form	
			$\frac{v\mathrm{d}v}{16-v^2}=0.625\mathrm{d}x$	M1	Separate variables	
			$-\frac{1}{2}\ln\left(16-v^2\right)=0.625x+c$	A1	Integrate both sides aef	
			$x=0, v=2:\ C=-\frac{1}{2}\ln(12)$	M1	Use condition	
			$-\frac{5}{4}x=\ln\left(\frac{16-v^2}{12}\right)$	M1	Rearrange to make v or v^2 the subject	
			$v=\sqrt{16-12\mathrm{e}^{-1.25x}}$	A1	cao nfww Note: finding v in terms of t: 0/6	
				[6]		
	(v)		Terminal speed is 4 $x=0.8\ln\frac{12}{7}=0.4311972\ldots$ so 0.431 (3 s.f.)	M1 A1 **[2]**	cao (accept exact answer)	
3.	**(a)**	**(i)**	IF: $\mathrm{e}^{-2\ln x}$	M1	Negative sign needed	
			$=\frac{1}{x^2}$	A1		
			$\frac{y}{x^2}=\int x\sin 2x\ \mathrm{d}x$	M1	Multiply through by their IF and recognise derivative on LHS	
			$=x\left[-\frac{\cos 2x}{2}\right]+\frac{1}{2}\int\cos 2x\ \mathrm{d}x$	M1	Use integration by parts	
			$=-\frac{1}{2}x\cos 2x+\frac{1}{4}\sin 2x+A$	M1 A1	Complete the integration cao	

Question			Answer	Marks	Guidance	
			GS: $y = x^2\left(A - \frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\right)$	F1 **[7]**	Divide their RHS by their IF	
		(ii)	$y = 0, x = \frac{1}{4}\pi$: $A = -\frac{1}{4}$ When $x = \frac{1}{2}\pi$, $y = \frac{\pi^2(\pi - 1)}{16}$	M1 A1 M1 A1 **[4]**	Use condition Substitute $x = \frac{1}{2}\pi$ cao	
		(iii)	$y_0' = 0.1487525$ One use of correct algorithm 0.0148752 0.0458240 0.09932	B1 M1 A1 A1 A1 **[5]**	 cao rounding correct to 3 s.f., as final answer	
	(b)	**(i)**	$y = x^2 - 1$: (parabola) (0, -1)	B1 B1 **[2]**		
		(ii)	Two correct isoclines All 5 correct isoclines Correct direction indicators	B1 B1 B1 **[3]**	 At least 3 on each isocline	
		(iii)	Attempt at one solution curve Curve through (0, 1) correct shape Curve through (1, 0) correct shape	M1 A1 A1 **[3]**	 Minimum in first quadrant Crossing negative y - axis	

Question			Answer	Marks	Guidance	
4.	(i)		$y'' + 9x' - y' = 7e^{7t}$	M1	Differentiate	
			$y'' + 9(7x + 2y + 13e^{4t}) - y' = 7e^{7t}$	M1	Substitute for x'	
			$x = \frac{1}{9}\left(y - y' + e^{7t}\right)$	M1	Rearrange	
			$y'' + 7(y - y' + e^{7t}) + 18y + 117e^{4t)}) - y' = 7e^{7t}$	M1	Substitute for x	
			$y'' - 8y' + 25y = -117e^{4t}$	A1		
			Auxiliary equation: $m^2 - 8m + 25 = 0$	M1		
			$m = 4 \pm 3i$	A1		
			CF: $y = e^{4t}\left(A\cos 3t + B\sin 3t\right)$	F1		
			PI: $y = Pe^{4t}$	B1	Correct form for their RHS	
			$y' = 4Pe^{4t}, y'' = 16Pe^{4t}$	M1	Differentiate, substitute and solve	
			$A = -13$	A1	cao	
			GS: $y = e^{4t}\left(A\cos 3t + B\sin 3t\right) - 13e^{4t}$	F1	CF with 2 arb constants + PI	
				[12]		
			Alternative scheme for finding differential equation in *x:*			
			$\ddot{x} = 7\dot{x} + 2\dot{y} + 52e^{4t}$	M1	Differentiate	
			$\ddot{x} = 7\dot{x} + 2(-9x + y + e^{7t}) + 52e^{4t}$	M1	Substitute for y'	
				M1	Rearrange and substitute for y	
			$\ddot{x} - 8\dot{x} + 25x = 39e^{4t} + 2e^{7t}$	A1	cao	
			Auxiliary eqn: $m^2 - 8m + 25 = 0$	M1		
			$m = 4 \pm 3i$	A1		
			CF: $x = e^{4t}(D\cos 3t + E\sin 3t)$	F1		
			PI: $x = Pe^{4t} + Qe^{7t}$	M1	Correct form for their RHS	
			$P = \frac{13}{3}, \ Q = \frac{1}{9}$	A1	cao	
			$x = e^{4t}(D\cos 3t + E\sin 3t) + \frac{13}{3}e^{4t} + \frac{1}{9}e^{7t}$			
			Differentiate	M1		
			Substitute and rearrange to find y	M1		
			$y = \frac{1}{2}e^{4t}(A\cos 3t + B\sin 3t) - 13e^{4t}$	A1	cao	

Question	Answer	Marks	Guidance
(ii)	$y=-3, t=0:\ A=10$ $y'=\mathrm{e}^{4t}\left(-3A\sin 3t+3B\cos 3t\right)-52\mathrm{e}^{4t}+4\mathrm{e}^{4t}(A\cos 3t+B\sin 2t)$ $y'=60, t=0:\quad B=24$ $y=\mathrm{e}^{4t}\left(10\cos 3t+24\sin 3t\right)-13\mathrm{e}^{4t}$	M1 M1 M1 A1 **[4]**	Use condition Differentiate using product rule Use condition cao
(iii)	Substitute for y and y' in $x=\frac{1}{9}\left(y-y'+\mathrm{e}^{7t}\right)$ $x=\frac{1}{9}\left(\mathrm{e}^{7t}+\mathrm{e}^{4t}\left(-102\cos 3t-42\sin 3t+39\right)\right)$	M1 A1 **[2]**	 cao oe (must be simplified)
(iv)	$\left(10\cos 3t+24\sin 3t\right)=13$ $\sin(3t+\alpha)=\frac{1}{2}$ or $\cos(3t-\alpha)=\frac{1}{2}$ where $\tan\alpha=\frac{5}{12}$ ($\alpha=0.394$) $t=\frac{\pi}{18}-\frac{1}{3}\tan^{-1}\left(\frac{5}{12}\right)$	M1 M1 A1 A1 **[4]**	Equate y to zero and attempt to solve Write in correct form or any other appropriate method $\tan\alpha=\frac{12}{5}$ if using cos formula, ($\alpha=1.176$) or 0.0429
(v)	Divide y and x by e^{7t} or e^{4t} and consider limit as $t\to\infty$ Limit is 0 AG	M1 E1 **[2]**	

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

Qualification Accredited

OCR
Oxford Cambridge and RSA

AS/A LEVEL GCE

Examiners' report

MATHEMATICS (MEI)

3895-3898, 7895-7898

4758/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4758/01 series overview

This was the final assessment series for the unitised 3895-3898, 7895-7898 GCE Mathematics (MEI) specification. There will be a resit opportunity in the summer 2019.

Differential Equations (DE) 4758 is an A2 GCE component taken as part of the Mathematics (MEI) specification, occasionally used as the second optional component in AS Further Mathematics (3896), but generally as one of the four optional components in A Level Further Mathematics (7896). A few candidates are also studying AS Further Mathematics (Additional) (3897) or A Level Further Mathematics (Additional) (7897), and in this case the grading optimisation process will determine which qualification this component will contribute towards.

Candidates are required to answer three of the four questions on this paper. Most candidates attempted Question 1 and Question 4 (Q1 98% and Q4 82% of candidates), and then one of the other two questions. Question 3 was the least popular (Q2 69% and Q3 52%). There were a small percentage of candidates that answered more than the required three questions: in these cases all questions were marked and the best three scores used. Generally, candidates are better advised to focus all their time on three specific questions rather than attempting all four questions in a time designed for three full answers; any time at the end would be better used to refine their earlier answers rather than attempting the extra question.

The general standard of the responses was high, with many candidates gaining full marks on more than one question. Candidates showed a sound knowledge of the methods and techniques involved. There were some arithmetical errors in basic algebra which proved costly for some candidates, but these appeared to be careless mistakes rather than an indication of any fundamental weakness; overall the standard of accuracy was very good.

Question 1(i)

1 In this question, you may assume that $t^k e^{-t} \to 0$ as $t \to \infty$ for any constant k.

The differential equation $4\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 9x = f(t)$ is to be solved for $t \geqslant 0$.

Firstly consider the case $f(t) = 9t^2 - 3t - 1$.

(i) Find the general solution for x in terms of t. **[9]**

Almost all candidates were able to apply the method for the solution of a second order differential equation. The forms of the complementary function and the particular integral were known and there were very few arithmetical errors.

Question 1(ii)

You are given that $x = 5$ and $\frac{dx}{dt} = 0$ when $t = 0$.

(ii) Find the particular solution. **[4]**

This was answered well with the majority of candidates scoring full marks.

Question 1(iii)

(iii) Show that x is positive for all values of $t \geqslant 0$. **[3]**

Candidates were required to show that both the exponential part and the quadratic part of the particular solution found in part (ii) were positive. This was done with varying degrees of success. The exponential term $(6t + 2)e^{-1.5t}$ is always positive for positive values of t. Some candidates seemed to think that the behaviour of the exponential function for large values of t was relevant here. There are several ways of showing that the quadratic term $t^2 - 3t + 3$ is positive: completing the square and noting that the constant term 0.75 is positive, or finding the turning point and noting that it was a positive quadratic with a minimum in the first quadrant, or using the discriminant to show that the quadratic does not cross the x–axis. Many candidates who adopted the last of these approaches did not realise that they needed to say why the curve was wholly above and not wholly below the axis.

Question 1(iv)

Now consider the case $f(t) = -48\sin 2t - 14\cos 2t$.

(iv) Find the general solution for x in terms of t. **[6]**

The majority of candidates earned full marks in this part. The only errors were arithmetical slips in solving the simultaneous equations when finding the particular integral.

Question 1(v)

(v) Describe the behaviour of x for large values of t. **[2]**

A comment on the behaviour of the exponential part of the solution found in part (iv), for large values of *t,* and a statement that the remaining part of the solution was oscillatory was required for the first mark. A second mark was credited for noting the amplitude of the resulting motion.

Question 2(i)

2 **Take g as 10 in this question.**

A particle P of mass 0.1 kg is in a liquid and is projected vertically downwards. At time t s, the velocity of P is $v\,\mathrm{m\,s^{-1}}$ and the depth of P below its point of projection, O, is x m. The only forces on P are its weight and a resistance force R N. A scientist investigates two different models for R.

In the first model, the resistance is given by $R = 0.2v$ and the initial speed of P is $2\,\mathrm{m\,s^{-1}}$.

(i) Use this information to form a differential equation involving v and t. Solve the differential equation to show that $v = 5 - 3\mathrm{e}^{-2t}$. **[7]**

Almost all candidates scored full marks in this part. The most popular method was by separation of variables.

Question 2(ii)

(ii) Sketch the graph of v against t. **[2]**

The majority of candidates successfully completing part (i) could provide an appropriate sketch, although a few candidates only scored one out of the two marks for not labelling either the x-intercept or the asymptote.

Question 2(iii)

(iii) Find an expression for x in terms of t and hence find the depth of P below O when its speed is three-quarters of its terminal speed. **[7]**

The majority answered this part well, although careless arithmetic errors prevented some candidates for progressing through to a successful conclusion.

Question 2(iv)

In the second model, the resistance is given by $R = 0.0625v^2$ and the initial speed of P is again $2\,\mathrm{m\,s^{-1}}$.

(iv) Find v in terms of x. **[6]**

The responses to this part were variable in quality. The best solutions showed a careful and accurate application of integration by separation of variables. A significant minority of candidates made little progress in the integration. Others attempted to find *v* in terms of *t*.

Question 2(v)

(v) State the terminal speed of P and find the depth of P below O when its speed is three-quarters of its terminal speed. **[2]**

Generally, those candidates that had progressed through the question could pick up at least one mark here, with around one half of the candidates gaining full marks.

Question 3(a)(i)

3 **(a)** A curve in the x-y plane satisfies the differential equation $\frac{dy}{dx} - \frac{2y}{x} = x^k \sin 2x$,

where k is a constant and $x > 0$.

Firstly consider the case $k = 3$.

(i) Find the general solution for y in terms of x. **[7]**

Responses to this part were almost always correct, although some careless arithmetic errors were seen. A few candidates did not multiply through the right-hand side by their correct integrating factor. This led to multiple applications of integration of parts, which did not deter them.

Question 3(a)(ii)

(ii) Given that $y = 0$ when $x = \frac{1}{4}\pi$, find the exact value of y when $x = \frac{1}{2}\pi$. **[4]**

The method was clearly known, but again there were many arithmetical errors that cost full credit.

Question 3(a)(iii)

Now consider the case $k = 2.5$.

(iii) Use Euler's method, with a step length of 0.1 and initial conditions $y = 0$ when $x = 0.5$, to estimate y when $x = 0.8$. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$. **[5]**

This was a routine request and the responses reflected this.

Question 3(b)(ii)

(ii) In your Answer Book, sketch on the given axes the isoclines for the cases $\frac{dy}{dx} = m$ for $m = 0, \pm 1, \pm 2$. Use these isoclines to draw a tangent field. **[3]**

For full marks, candidates needed to sketch five isoclines, each one a parabola, and mark on each one a few direction indicators. Some candidates attempted to mark direction indicators without sketching the isoclines. Other candidates sketched only two or three of the isoclines. A common error was to assume that a parabola with a positive y-intercept had positive direction indicators.

Question 3(b)(iii)

(iii) Sketch the solution curve through (0, 1) and the solution curve through (1, 0). **[3]**

There were some very pleasing sketches of the two solution curves. A common error was to have the reflections of the correct curves, in the *y*-axis. This error usually resulted from incorrect direction indicators in part (ii).

Question 4(i)

4 The simultaneous differential equations

$$\frac{dx}{dt} = 7x + 2y + 13e^{4t},$$

$$\frac{dy}{dt} = -9x + y + e^{7t}$$

are to be solved.

(i) Eliminate x to obtain a second order differential equation for y in terms of t. Hence find the general solution for y. **[12]**

This is a standard question on the solution of simultaneous differential equations, but with a slight twist. The instruction to eliminate *x* to obtain a second order differential equation for *y* in terms of *t* was less familiar to candidates and many embarked on the more familiar route of eliminating *y* to find a differential equation for *x* in terms of *t*. Having realised their error, the majority of candidates who had pursued this route crossed out their work and began again. Others abandoned the question altogether. A very small minority realised that they could use their solution for *x* in the second given differential equation and thereby find *y*.

Question 4(ii)

(ii) Given that $y = -3$ and $\frac{dy}{dt} = 60$ when $t = 0$, find the particular solution for y. **[4]**

Careless arithmetic in the substitutions prevented candidates from gaining full credit

Question 4(iii)

(iii) Find the corresponding particular solution for x. **[2]**

Over half of the candidates gained the first method mark, but errors in the algebraic manipulation resulted in candidates missing out on the accuracy mark.

Question 4(iv)

(iv) Find the smallest positive value of t for which $y = 0$. **[4]**

Most candidates equated their solution to part (ii) to zero, but only a minority were able to solve the resulting trigonometric equation: $10\cos 3t + 24\sin 3t = 13$. The most common method of solution was to equate the right-hand side to $R\sin(3t + \alpha)$.

Question 4(v)

(v) Show that $\frac{y}{x} \to 0$ as $t \to \infty$. **[2]**

Only a minority of candidates were able to make any convincing progress in this part. Many of the arguments developed were too vague to be awarded full credit.

Supporting you

For further details of this qualification please visit the subject webpage.

Review of results

If any of your students' results are not as expected, you may wish to consider one of our review of results services. For full information about the options available visit the OCR website. If university places are at stake you may wish to consider priority service 2 reviews of marking which have an earlier deadline to ensure your reviews are processed in time for university applications.

activeresults

Active Results offers a unique perspective on results data and greater opportunities to understand students' performance.

It allows you to:

- Review reports on the **performance of individual candidates**, cohorts of students and whole centres
- **Analyse results** at question and/or topic level
- **Compare your centre** with OCR national averages or similar OCR centres.
- Identify areas of the curriculum where students excel or struggle and help **pinpoint strengths and weaknesses** of students and teaching departments.

http://www.ocr.org.uk/administration/support-and-tools/active-results/

CPD Hub
Your route to OCR's teacher training

Attend one of our popular CPD courses to hear exam feedback directly from a senior assessor or drop in to an online Q&A session.

https://www.cpdhub.ocr.org.uk

The small print

We'd like to know your view on the resources we produce. By clicking on the 'Like' or 'Dislike' button you can help us to ensure that our resources work for you. When the email template pops up please add additional comments if you wish and then just click 'Send'. Thank you.

Whether you already offer OCR qualifications, are new to OCR, or are considering switching from your current provider/awarding organisation, you can request more information by completing the Expression of Interest form which can be found here: www.ocr.org.uk/expression-of-interest

OCR Resources: *the small print*
OCR's resources are provided to support the delivery of OCR qualifications, but in no way constitute an endorsed teaching method that is required by OCR. Whilst every effort is made to ensure the accuracy of the content, OCR cannot be held responsible for any errors or omissions within these resources. We update our resources on a regular basis, so please check the OCR website to ensure you have the most up to date version.

This resource may be freely copied and distributed, as long as the OCR logo and this small print remain intact and OCR is acknowledged as the originator of this work.

Our documents are updated over time. Whilst every effort is made to check all documents, there may be contradictions between published support and the specification, therefore please use the information on the latest specification at all times. Where changes are made to specifications these will be indicated within the document, there will be a new version number indicated, and a summary of the changes. If you do notice a discrepancy between the specification and a resource please contact us at: resources.feedback@ocr.org.uk.

OCR acknowledges the use of the following content:
Square down and Square up: alexwhite/Shutterstock.com

Please get in touch if you want to discuss the accessibility of resources we offer to support delivery of our qualifications: resources.feedback@ocr.org.uk

Looking for a resource?

There is now a quick and easy search tool to help find **free** resources for your qualification:

www.ocr.org.uk/i-want-to/find-resources/

www.ocr.org.uk

OCR Customer Contact Centre

General qualifications
Telephone 01223 553998
Facsimile 01223 552627
Email general.qualifications@ocr.org.uk

OCR is part of Cambridge Assessment, a department of the University of Cambridge. *For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored.*

Cambridge Assessment

CERTIFIED ISO 9001 UKAS MANAGEMENT SYSTEMS 001

Qualification Accredited

OCR
Oxford Cambridge and RSA

AS/A LEVEL GCE

Examiners' report

MATHEMATICS (MEI)

3895-3898, 7895-7898

4758/02 Summer 2018 series

Moderated component

Version 1

Contents

Introduction

The coursework component of Differential Equations 4758 aimed to provide candidates with the opportunity to undertake the various steps of problem solving, comparing theoretical values predicted using modelling to experimental data. Candidates could approach the coursework in two different ways.

(A) In this case the modelling cycle is investigated in some depth, whilst the check against reality may use the data from published sources, from experiments which the candidate has not actually performed or from experience; there must however be a quantitative element in such data.

(B) The work presented is approximately evenly divided between developing the model, and one or more experiments conducted by the candidate to verify the quality of predictions from it and/or to inform its development.

OCR hopes that you and your candidates have enjoyed working through the coursework task. Although not part of the reformed A Level Further Mathematics assessment structure, you may find this task a useful teaching and learning activity when covering the Differential Equations content under DfE criteria reference I1-I9.
https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/516950/GCE_AS_and_A_level_subject_content_for_further_mathematics_with_appendices.pdf

Resitting Differential Equations

This was the final year of the unitised A Level Further Maths qualification 7896. There is a resit series in 2019 for those candidates that have certificated 7896 in a previous year. Those candidates that wish to resit 4758 may choose to carry over their coursework mark and only sit the examination paper, or to undertake a new coursework task and sit the examination paper (it is not possible to only complete a new coursework task and carry over the previous examination mark).

The entry code 4758B should be used for those candidates that wish to carry over their coursework task.

The entry code 4758A should be used for those candidates that wish to complete a new coursework task. Please note that these candidates are not permitted simply to improve a previous, marked, piece of work; they must undertake a new coursework task.

Administration

It was pleasing to note that moderators reported that most centres had administered the coursework efficiently, meaning that marks were submitted on time and the sample request dealt with speedily. This all made the process of external moderation very much easier.

In a very small number of instances, marks were not submitted until many days after the deadline set by OCR and the sample sent very late. This causes pressure on moderators who have less time to do their work. There were few clerical errors, and inconsistent marking resulting in an invalid order of merit occurred only in a handful of centres.

The major problem for moderators is caused when assessors tick work that has not been checked and in a number of cases credit was being given for incorrect work.

The marks of candidates in most centres were appropriate and acknowledgement is made of the amount of work that this involves to mark and internally moderate. The component specific comments are offered for the sake of centres who have had their marks adjusted for some reason. Centres should note that having marks adjusted does not imply inconsistent marking, merely that the level has been misjudged.

Common issues

When investigating modelling tasks it should be appreciated that the criteria in Domains 2 and 4 only apply to the initial model. While there may well be, for example, variation considered for the revised model, this cannot be used to fulfil the criteria in these domains.

Although there was a range of coursework tasks presented, as in previous years *Aeroplane Landing* and *Cascades* are still the most popular choices for many candidates. In the case of the former, work is still being produced where the initial model is rejected on the basis of the motion in the first 9 seconds, without considering the effect of the braking motion. This should be penalised. Similarly, in the latter case the focus of *'Cascades'* has to be on the flow through the second container. The initial model should not be rejected on the basis of the flow through a single container. This should be used in order to calculate the necessary parameter.

It is also worth mentioning that, particularly as the Differential Equations are given for the *'Interacting Species'*, a very clear description of the parameters is expected.

The essential function of the coursework element of this module is to test the candidates' ability to follow the modelling cycle. That is, setting up a model, testing it and then modifying the assumptions to improve the original model. If two or three models are suggested at the outset and tested, more or less simultaneously, and the best chosen, then the modelling cycle has not been followed.

Finally, it is usually easier to agree with those assessors that provide the most detailed signposting, through comments and references, of where and why marks were credited.

Supporting you

For further details of this qualification please visit the subject webpage.

Review of results

If any of your students' results are not as expected, you may wish to consider one of our review of results services. For full information about the options available visit the OCR website. If university places are at stake you may wish to consider priority service 2 reviews of marking which have an earlier deadline to ensure your reviews are processed in time for university applications.

activeresults

Active Results offers a unique perspective on results data and greater opportunities to understand students' performance.

It allows you to:

- Review reports on the **performance of individual candidates**, cohorts of students and whole centres
- **Analyse results** at question and/or topic level
- **Compare your centre** with OCR national averages or similar OCR centres.
- Identify areas of the curriculum where students excel or struggle and help **pinpoint strengths and weaknesses** of students and teaching departments.

http://www.ocr.org.uk/administration/support-and-tools/active-results/

CPD Hub
Your route to OCR's teacher training

Attend one of our popular CPD courses to hear exam feedback directly from a senior assessor or drop in to an online Q&A session.

https://www.cpdhub.ocr.org.uk

The small print

We'd like to know your view on the resources we produce. By clicking on the 'Like' or 'Dislike' button you can help us to ensure that our resources work for you. When the email template pops up please add additional comments if you wish and then just click 'Send'. Thank you.

Whether you already offer OCR qualifications, are new to OCR, or are considering switching from your current provider/awarding organisation, you can request more information by completing the Expression of Interest form which can be found here: www.ocr.org.uk/expression-of-interest

OCR Resources: *the small print*
OCR's resources are provided to support the delivery of OCR qualifications, but in no way constitute an endorsed teaching method that is required by OCR. Whilst every effort is made to ensure the accuracy of the content, OCR cannot be held responsible for any errors or omissions within these resources. We update our resources on a regular basis, so please check the OCR website to ensure you have the most up to date version.

This resource may be freely copied and distributed, as long as the OCR logo and this small print remain intact and OCR is acknowledged as the originator of this work.

Our documents are updated over time. Whilst every effort is made to check all documents, there may be contradictions between published support and the specification, therefore please use the information on the latest specification at all times. Where changes are made to specifications these will be indicated within the document, there will be a new version number indicated, and a summary of the changes. If you do notice a discrepancy between the specification and a resource please contact us at: resources.feedback@ocr.org.uk.

OCR acknowledges the use of the following content:
Square down and Square up: alexwhite/Shutterstock.com

Please get in touch if you want to discuss the accessibility of resources we offer to support delivery of our qualifications: resources.feedback@ocr.org.uk

Looking for a resource?

There is now a quick and easy search tool to help find **free** resources for your qualification:

www.ocr.org.uk/i-want-to/find-resources/

www.ocr.org.uk

OCR Customer Contact Centre

General qualifications
Telephone 01223 553998
Facsimile 01223 552627
Email general.qualifications@ocr.org.uk

OCR is part of Cambridge Assessment, a department of the University of Cambridge. *For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored.*

Cambridge Assessment

CERTIFIED ISO 9001 UKAS MANAGEMENT SYSTEMS 001

Unit level raw mark and UMS grade boundaries June 2018 series

For more information about results and grade calculations, see https://www.ocr.org.uk/students/getting-your-results/

AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

AS & Advanced GCE Mathematics

				Max Mark	a	b	c	d	e	u
4721	01	C1 Core mathematics 1 (AS)	Raw	72	61	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4722	01	C2 Core mathematics 2 (AS)	Raw	72	55	49	43	37	31	0
			UMS	100	80	70	60	50	40	0
4723	01	C3 Core mathematics 3 (A2)	Raw	72	55	48	41	34	28	0
			UMS	100	80	70	60	50	40	0
4724	01	C4 Core mathematics 4 (A2)	Raw	72	54	47	40	34	28	0
			UMS	100	80	70	60	50	40	0
4725	01	FP1 Further pure mathematics 1 (AS)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4726	01	FP2 Further pure mathematics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4727	01	FP3 Further pure mathematics 3 (A2)	Raw	72	47	41	36	31	26	0
			UMS	100	80	70	60	50	40	0
4728	01	M1 Mechanics 1 (AS)	Raw	72	60	51	42	34	26	0
			UMS	100	80	70	60	50	40	0
4729	01	M2 Mechanics 2 (A2)	Raw	72	53	46	39	32	26	0
			UMS	100	80	70	60	50	40	0
4730	01	M3 Mechanics 3 (A2)	Raw	72	50	42	34	27	20	0
			UMS	100	80	70	60	50	40	0
4731	01	M4 Mechanics 4 (A2)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4732	01	S1 – Probability and statistics 1 (AS)	Raw	72	57	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4733	01	S2 – Probability and statistics 2 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4734	01	S3 – Probability and statistics 3 (A2)	Raw	72	59	50	41	32	24	0
			UMS	100	80	70	60	50	40	0
4735	01	S4 – Probability and statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4736	01	D1 – Decision mathematics 1 (AS)	Raw	72	55	48	42	36	30	0
			UMS	100	80	70	60	50	40	0
4737	01	D2 – Decision mathematics 2 (A2)	Raw	72	58	53	48	44	40	0
			UMS	100	80	70	60	50	40	0

OCR
Oxford Cambridge and RSA

AS & Advanced GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – Introduction to advanced mathematics (AS)	Raw	72	60	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – Concepts for advanced mathematics (AS)	Raw	72	53	47	41	36	31	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) Methods for Advanced Mathematics (A2): Written Paper	Raw	72	61	56	51	46	40	0
4753	02	(C3) Methods for Advanced Mathematics (A2): Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) Methods for Advanced Mathematics (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – Applications of advanced mathematics (A2)	Raw	90	63	56	49	43	37	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – Further concepts for advanced mathematics (AS)	Raw	72	55	51	47	43	40	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – Further methods for advanced mathematics (A2)	Raw	72	48	42	36	31	26	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – Further applications of advanced mathematics (A2)	Raw	72	63	56	49	42	35	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) Differential Equations (A2): Written Paper	Raw	72	61	54	48	42	35	0
4758	02	(DE) Differential Equations (A2): Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) Differential Equations (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – Mechanics 1 (AS)	Raw	72	51	44	37	31	25	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – Mechanics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – Mechanics 3 (A2)	Raw	72	61	54	48	42	36	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – Mechanics 4 (A2)	Raw	72	59	51	44	37	30	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – Statistics 1 (AS)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – Statistics 2 (A2)	Raw	72	54	47	41	35	29	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – Statistics 3 (A2)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – Decision mathematics 1 (AS)	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – Decision mathematics 2 (A2)	Raw	72	55	51	47	43	39	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) Numerical Methods (AS): Written Paper	Raw	72	57	52	48	44	39	0
4776	02	(NM) Numerical Methods (AS): Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) Numerical Methods (AS): Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

AS GCE Statistics (MEI)

				Max Mark	a	b	c	d	e	u
G241	01	Statistics 1 MEI	Raw	72	No entry in June 2018					
			UMS	100	80	70	60	50	40	0
G242	01	Statistics 2 MEI	Raw	72	No entry in June 2018					
			UMS	100	80	70	60	50	40	0
G243	01	Statistics 3 MEI	Raw	72	No entry in June 2018					
			UMS	100	80	70	60	50	40	0

AS GCE Quantitative Methods (MEI)

				Max Mark	a	b	c	d	e	u
G244	01	Introduction to Quantitative Methods (Written Paper)	Raw	72	58	50	43	36	28	0
G244	02	Introduction to Quantitative Methods (Coursework)	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
G245	01	Statistics 1	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
G246	01	Decision Mathematics 1	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0

Level 3 Certificate, Level 3 Extended Project and FSMQ raw mark grade boundaries June 2018 series

For more information about results and grade calculations, see https://www.ocr.org.uk/students/getting-your-results/

Level 3 Certificate Mathematics - Quantitative Methods (MEI)												
					Max Mark	a	b	c	d	e	u
G244	A	01	Introduction to Quantitative Methods with Coursework (Written Paper)	Raw	72	58	50	43	36	28	0
G244	A	02	Introduction to Quantitative Methods with Coursework (Coursework)	Raw	18	14	12	10	8	7	0
				UMS	100	80	70	60	50	40	0
				Overall	90	72	62	53	44	35	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI)											
				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H866	02	Critical maths	Raw	60	44	39	34	29	24	0
		*To create the overall boundaries, component 02 is weighted to give marks out of 72	Overall	144	109	96	83	70	57	0

Level 3 Certificate Mathematics - Quantitative Problem Solving (MEI)										
				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H867	02	Statistical problem solving	Raw	60	40	36	32	28	24	0
		*To create the overall boundaries, component 02 is weighted to give marks out of 72	Overall	144	104	92	80	69	57	0

Advanced Free Standing Mathematics Qualification (FSMQ)										
				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	56	50	44	38	33	0

Intermediate Free Standing Mathematics Qualification (FSMQ)										
				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0